Trapping, compression, and acceleration of an electron bunch in the nonlinear laser wakefield

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A scheme of laser wakefield acceleration, when a relatively rare and long bunch of nonrelativistic or weakly relativistic electrons is initially in front of the laser pulse, is suggested and considered. The motion of test electrons is studied both in the one-dimensional (1D) case (1D wakefield) and in the case of three-dimensional laser wakefield excited in a plasma channel. It is shown that for definite parameters of the problem the bunch can be trapped, effectively compressed both in longitudinal and transverse directions, and accelerated to ultra-relativistic energies in the region of first accelerating maximum of the wakefield. The accelerated bunch has sizes much less than the plasma wavelength and relatively small energy spread.

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I. INTRODUCTION

The rapid progress in the technology of high-intensity lasers, based on the chirped-pulse amplification (CPA) [1], opens opportunities for the use of lasers in many branches of science and industry. Relatively inexpensive tabletop terawatt lasers (so-called T^3 lasers) become a tool in physical researches and now are available at many laboratories over the world. Last year's CPA technique permitted the production of subpicosecond laser pulses of multiterawatt power with peak intensities exceeding 10^{20} W/cm^2 [2]. With intensities as such we practically have to do with an interaction range of laser radiation with matter, where the role of the nonlinear effects is often essential. In this intense laser field the matter is usually transformed to plasma and free electrons oscillate with relativistic quivering energy. Presently, the interactions of high-power laser radiation with plasma are actively investigated in connection with different applications: the excitation of strong plasma wake waves for focusing and acceleration of charged bunches [3]; generation of radiation at harmonics of carrier laser frequency [4]; x-ray sources [5]; laser inertial fusion [6]; etc.

The laser wakefield, generated in plasma by the short (with the length $\approx\lambda_p/2$, where λ_p is the plasma wavelength) intense laser pulse provides the acceleration gradient up to tens GeV/m (laser wakefield acceleration, LWFA [7,8]), that is three orders of magnitude higher than that achieved in conventional accelerators. The main aim of experimental and theoretical works, that are presently in progress, is the construction of compact and relatively inexpensive accelerators of charged particles for applications in physics research, medicine and hi-tech industry. However, some challenges remain on this way, the main one of those is the problem of electron bunch injection.

The wake wavelength in the LWFA is $\lambda_p\approx 2c\tau_L$ [3] (here τ_L is the laser-pulse duration) and makes up tens or hundreds micrometers for typical plasma densities $n_p\sim 10^{16}-10^{19}$ cm^{-3}. To obtain high-quality relativistic electron bunch accelerated by the wake wave, it is necessary to inject short (with the length $L\ll\lambda_p$), enough dense relativistic electron bunch in the accelerating phase of the wake wave with femtosecond synchronization, that is a difficult technical problem (see, e.g., Ref. [9]). The injection schemes proposed for the standard LWFA (the LIPA [9], the colliding laser pulses [10], and the LILAC [11] schemes) are aimed at the generation of such a short relativistic bunch.

The diffraction broadening leads to a rapid decrease of the intense laser-pulse amplitude with the characteristic length $Z_R=\pi r_0^2/\lambda_L$ (here Z_R is the Rayleigh length, r_0 is the focal spot size of the pulse, and λ_L is the laser wavelength) that is typically in order of a millimeter. To prevent diffraction, the plasma channel with minimum density at the axis, proposed to guide the laser pulse in LWFA [12]. The amplitude of the accelerating component of the wake wave generated in the plasma channel decreases as distance from the laser pulse increases [13,14]. Besides, the change of the plasma wavelength λ_p in the transverse direction ($\lambda_p\sim[n_p(r)]^{-1/2}$, where n_p is density of electrons in the plasma channel), leads to undesirable wave-front curving; this effect becomes stronger as distance from the pulse increases. The effect of wave-front curving in the channel, in the case of a strong laser pulse ($a_0=eE_m/m_ec\omega_L\geqslant 1$, where e and m_e are the absolute charge and mass of the electron, E_m is the maximum amplitude of laser field, c is the speed of light in vacuum, and ω_L is the laser frequency), is amplified due to the nonlinear change of the wake wavelength in transverse direction [15,16]. Thus, for regular acceleration of a charged bunch in the wake wave, most preferable is the region of the first maximum of accelerating field behind the laser pulse.

To avoid the aforesaid difficulties in LWFA, we suggest and study in this paper a scheme of trapping, compression, and acceleration of a nonrelativistic or weakly relativistic electron bunch in the laser wakefield, when the bunch is initially in front of the laser pulse. The initial bunch density can be much less than that required for accelerating bunch and the bunch sizes can be in order or more than the plasma wavelength, i.e., much more than required other by methods of injection [9–11]. Our investigations take into account both the pulse ponderomotive force and the wakefield. It is shown that for definite parameters of the problem the bunch can be trapped, effectively compressed both in longitudinal and transverse directions, and accelerated to ultrarelativistic energies in the region of first accelerating the maximum of the wakefield. The accelerated bunch has sizes much less than the plasma wavelength and enough good quality.

II. THE CASE OF WIDE LASER PULSE

At first we neglect the transverse variation of the laser-pulse amplitude and consider the case of one-dimensional laser wakefield excited by a wide pulse in uniform plasma. This allows us to study the longitudinal dynamics of the bunch electrons in more details.

A. Basic equations and correlations

The one-dimensional steady wakefield excited by a linearly polarized laser pulse is described by the following equation (see, e.g., Ref. [3]):

$$\frac{d^2\Phi}{d\xi^2}+\beta_g\gamma_g^2\left\{1-\beta_g\frac{\Phi/(1+a^2/2)^{1/2}}{[\Phi^2/(1+a^2/2)-\gamma_g^{-2}]^{1/2}}\right\}=0, \quad (1)$$

where $\Phi=1+e\varphi/m_ec^2$ is the dimensionless potential of the plasma wakefield, $a=eE_0(\xi)/m_ec\omega_L$, E_0 is the electric-field amplitude of the laser pulse, $\xi=k_p(z-v_gt)$, $k_p=\omega_p/v_g$, $\omega_p=(4\pi n_pe^2/m_e)^{1/2}$ is the plasma frequency, v_g is the group velocity of the laser pulse which is equal to the phase velocity of the wake wave, $\beta_g=v_g/c$, $\gamma_g=(1-\beta_g^2)^{-1/2}$ is the relativistic factor, which, in the case $\gamma_g\gg1$, is nearly equal to ω_L/ω_p. The electric field of exited wakefield, normalized to the nonrelativistic wave-breaking field $E_{WB}=m_ev_g\omega_p/e$, can be found from the equation $E_z=-(1/\beta_g)^2d\Phi/d\xi$. The equation of motion for a test electron in the wakefield and in the laser pulse field is (see, e.g., Ref. [17])

$$\frac{dp}{d\tau}=-\frac{1}{4\beta_g\gamma}\frac{da^2}{d\xi}-\beta_gE_z. \quad (2)$$

Here the first term on the right-hand side is the relativistic ponderomotive force averaged over the fast laser oscillations, and the second one corresponds to the plasma wakefield excited by the laser pulse, $\beta=v/c$, $p=\beta\gamma$, and $\gamma=(1+p^2+a^2/2)^{1/2}=[(1+a^2/2)/(1-\beta^2)]^{1/2}$ are the normalized longitudinal velocity and momentum and the relativistic factor of the test electron (transverse velocity is zero in this section), $\tau=\omega_pt$ is the normalized time. Multiplying Eq. (2) by β, one obtains after simple mathematics the following integral of motion (see also Refs. [14,18]):

$$\gamma-\beta_gp-\Phi=\text{const}. \quad (3)$$

Let us consider an electron which is initially situated ahead of the laser pulse at a certain point ξ_0, where $\Phi=1$ and $a=0$. If the electron has initial momentum $p_0<\beta_g\gamma_g$, it will be overtaken by the laser pulse and can be trapped at some point ξ_r inside the pulse or in the wake and accelerated. At the trapping point (or, in other words, at the point of reflection) the electron velocity becomes equal to v_g. Then from expression (3) we have

$$S\equiv(1+a_r^2/2)^{1/2}/\gamma_g-(\Phi_r-1)=(1+p^2)^{1/2}-\beta_gp. \quad (4)$$

In Eq. (4) a_r and Φ_r are the pulse amplitude and the wakefield potential at the reflection point ξ_r. From Eq. (4) one has

$$p=\gamma_g[\beta_g\gamma_gS\pm(\gamma_g^2S^2-1)^{1/2}]. \quad (5)$$

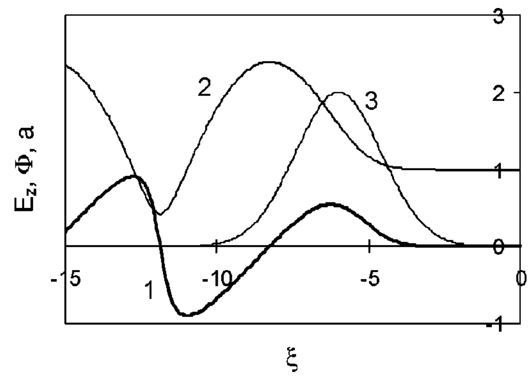

FIG. 1. The one-dimensional nonlinear wakefield excited by the linearly polarized laser pulse with peak normalized amplitude $a_0=2$, $\sigma_z=2$, $\gamma=10$. 1, the longitudinal electric field $E_z(\xi)$; 2, the potential of the wakefield $\Phi(\xi)$; 3, the amplitude of the laser pulse $a(\xi)$. All variables are normalized (see the text).

The minus sign in Eq. (5) corresponds to the initial momentum p_0 of an electron which has momentum $\beta_g\gamma_g$ at the point ξ_r and the plus sign corresponds to the momentum the of free electron which initially was at the point ξ_r. Expression (5) describes both trapped and passing particles. In the wake, electrons can be trapped only in the region where $E_z\leqslant0$. Equation of motion (2) can be rewritten in the form [19]

$$\frac{d^2\xi}{d\tau^2}+\frac{(1-\beta_g\beta)}{4\beta_g^2\gamma^2}\frac{da^2}{d\xi}+\frac{(1-\beta^2)}{\gamma}E_z=0, \quad (6)$$

where ξ is the normalized coordinate of a test electron in the frame comoving with the laser pulse. The dimensionless velocity of the electron one can obtain from the expression $\beta=\beta_g(1+d\xi/d\tau)$.

B. Numerical results

Equations (1) and (6) were solved numerically for the Gaussian laser pulse

$$a=a_0\exp[-(\xi-\xi_c)^2/\sigma_z^2].$$

A laser pulse with $a_0=2$ and nonlinear wakefield excited by it are presented in Fig. 1 (here and below in numerical calculations $\sigma_z=2$, $\xi_c=3\sigma_z$, and $\gamma_g=10$). The amplitude of the wake wave is essentially less than one-dimensional relativistic wave-breaking field $E_{rel}=[2(1-\gamma_g)]^{1/2}/\beta_g\approx4.26$ [3,20]. Figure 2 shows the dependence of initial electron momentum p_0 on the trapping point near the first accelerating maximum in the wake wave. The minimum value of the initial momentum $p_{\min}$ corresponds to the trapping point, where the potential achieves its minimum and $E_z=0$. Curves 1 and 2 in Fig. 2 reach their minima at different points, that is the consequence of the nonlinear increase of wake wavelength with increasing amplitude (the dependence of the wavelength on the amplitude can be found in Ref. [21]). The curves were obtained numerically and coincide with the expression (5) for the trapped particles. Figure 3 shows the dependence of the value of $p_{\min}$ and wake wave amplitude $E_{z,\max}$ on a_0. One can see that the laser pulse with $a_0\sim1$

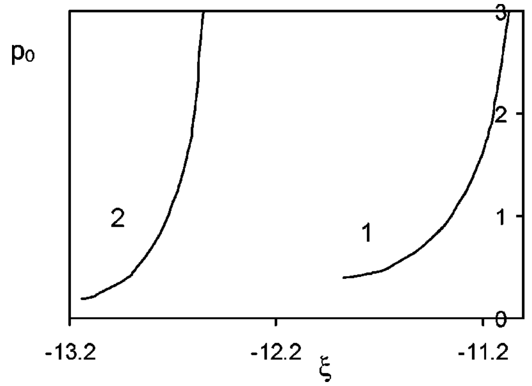

FIG. 2. Dependence of dimensionless electron initial momentum p_0 on the trapping point near the first accelerating maximum. 1, $a_0=2$; 2, $a_0=3$.

(that corresponds to the pulse peak intensity $I_{\max}\sim 10^{18}$ W/cm^2 at the laser wavelength $\lambda_L=1\,\mu$m, and $I_{\max}\sim 10^{16}$ W/cm^2 at $\lambda_L=10\ \mu$m) provides trapping of initially nonrelativistic or weakly relativistic electrons in the wake wave. For instance, $p_{\min}\approx 0.4$ for the wakefield presented in Fig. 1. Electrons with $p_0<p_{\min}$ are not trapped in the wake wave and can be detected behind the wave. This fact may be used to determine the wake wave amplitude in experiments from the $p_{\min}(E_{z,\max})$ dependence (Fig. 3). Our numerical calculations have witnessed that electrons with $p_0<\beta_g\gamma_g$ cannot be trapped in the region of laser pulse because of the decelerating wakefield; only the electrons with $p_0\approx\beta_g\gamma_g$ can be trapped by the leading edge of the pulse (where $E_z\approx 0$) due to the ponderomotive force. This confirms the results of Ref. [18]. In the case of long laser pulse ($\tau_L>\omega_p^{-1}$) the wakefield can be essentially reduced and direct laser ponderomotive acceleration of preaccelerated electrons is possible [22]. When the pulse is short ($\tau_L\sim\omega_p^{-1}$), plasma tends to compensate for the laser ponderomotive force, so that the decelerating wakefield is strong enough to prevent the trapping and effective ponderomotive acceleration of electrons inside the pulse (see Fig. 1).

Figure 4 shows the behavior of electrons of monoenergetic nonrelativistic [$p_0=0.5$, $\gamma_0=(1+p_0^2)^{1/2}\approx 1.12$] bunch in the wakefield presented in Fig. 1. The initial dimensionless bunch length $L_0=5$ corresponds, approximately, to the linear plasma wavelength λ_p. For $\tau=50$, the trapped bunch length is $L\approx 0.027$ and $L\approx 0.04$ for $\tau=100$, that is two orders of magnitude less than the initial bunch length. The absolute energy spread $\Delta\gamma$ in the accelerating bunch increases insignificantly with time, but the relative energy spread $\varepsilon=\Delta\gamma/\gamma$ falls due to growing γ; for example, $\varepsilon\approx 0.26$ at $\tau=50$, and $\varepsilon\approx 0.14$ at $\tau=100$. The acceleration gradient in the considering case is approximately equal to 2 MeV/λ_p. For example, when $\lambda_p=100\ \mu$m ($n_p\approx 10^{17}$ cm^{-3}), the acceleration gradient is 20 GeV/m.

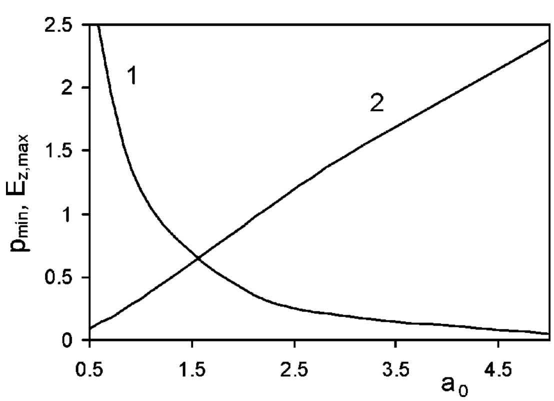

FIG. 3. The minimum dimensionless momentum of the trapped electrons $p_{\min}$ (curve 1) and the normalized wake wave amplitude $E_{z,\max}$ (curve 2) in dependence on peak amplitude of the laser pulse a_0.

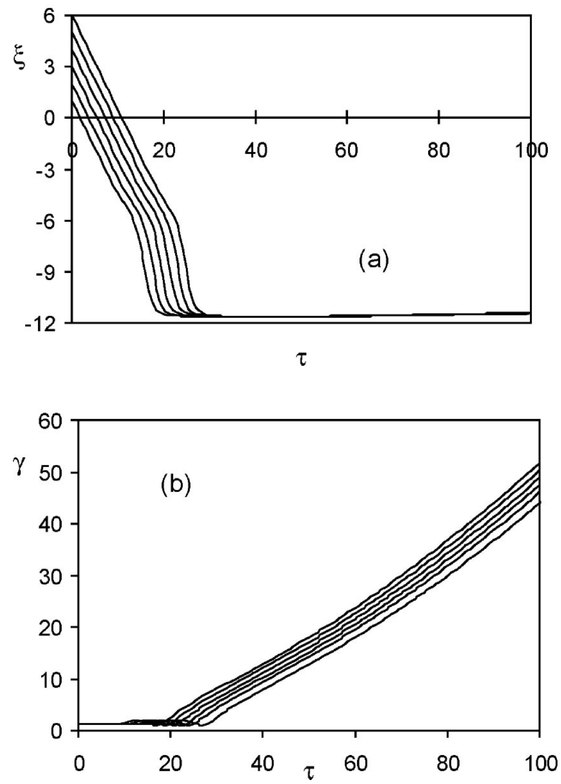

FIG. 4. Trapping, compression, and acceleration of the initially monoenergetic electron bunch in the wakefield presented in Fig. 1, $p_0=0.5$, $1\leqslant\xi_0\leqslant 6$. Evolution of the coordinate (a) and the relativistic factor (b) of electrons. The coordinate and time are normalized.

Figure 5 illustrates the motion of electrons with different initial momenta and the same initial positions ($0.6\leqslant p_0\leqslant 1.2$, $1.17\leqslant\gamma\leqslant 1.56$, $\xi_0=0$) in the wakefield presented in Fig. 1. The trapped bunch length is nearly 27 times less than the plasma wavelength λ_p. The relative energy spread at $\tau=100$ is lower than 0.1, that is much less than the spread in the initial electron bunch.

The dephasing length, for electrons with $p_{\min}\leqslant p_0\leqslant 1.2$, varies in the range $630\leqslant L_d\leqslant 700$ (the greater values correspond to the smaller initial momenta) that is comparable with the linear dephasing length $\lambda_p\gamma_g^2$ [3] (which, in our notations, corresponds to $L_d=2\pi\gamma_g^2=200\pi$). The maximum relativistic factor of accelerated particles varies in the range $350\leqslant\gamma_{\max}\leqslant 410$ (here again the greater values correspond to

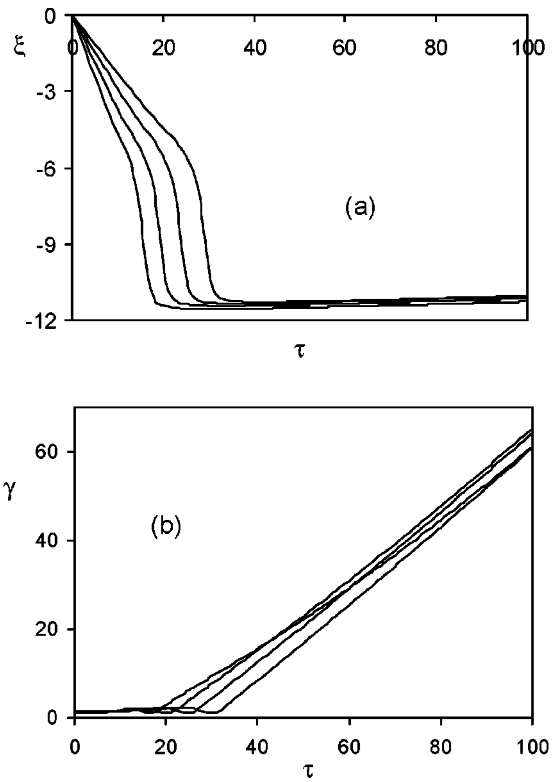

FIG. 5. Behavior of electrons with initial position $\xi_0=0$ and with initial momenta $p_0=0.5$, 0.8, 1, and 1.2 in the wakefield shown in Fig. 1. Electrons with smaller initial momenta are trapped earlier. (a) The normalized coordinate and (b) relativistic factor of electrons.

smaller p_0), that essentially exceeds the linear value $2\gamma_g^2=200$ [3], but is an order of magnitude less than the maximum nonlinear value $4\gamma_g^3=4000$ [21,23].

C. Energy spread in accelerating bunch

The energy spread in the trapped bunch depends both on energy spread and length of the initial bunch. The tail electrons of the initial bunch are trapped earlier and therefore, have greater energy at a given τ during acceleration (see Fig. 4). Slower particles also are trapped earlier (see Fig. 5). Let us suppose that the bunch is initially situated ahead of the laser pulse, so that $\xi=0$ corresponds to the bunch tail, and $\tau_{tr}(p_0)$ is the time necessary to trap an electron which is initially at $\xi=0$; the initial electron momentum is in the range $p_1\leqslant p_0\leqslant p_2$. Then, for the energy spread in the trapped bunch one can write $\Delta\gamma\sim\Delta\tau_{tr}E_{z,\max}=[\tau_{tr}(p_2)-\tau_{tr}(p_1)+L_0/(1-v_2/v_g)]E_{z,\max}$, where $\Delta\tau_{tr}$ is the time interval which is necessary to trap the initial bunch. For the relative energy spread one has $\varepsilon\sim\Delta\tau_{tr}/(\tau-\Delta\tau_{tr})$. These estimates agree well with the numerical results. One can see that the presence of fast electrons (with $v_0\sim v_g\approx c$) in the initial bunch leads to an undesirable increase in the energy spread.

The trapped bunch density can be found from expression $n_b(\tau)\approx n_{b0}L_0/L(\tau)$, where n_{b0} is the initial bunch density.

D. Wakefield generated by accelerating bunch

The trapped bunch also generates wakefield which can destroy the laser wakefield and decrease the accelerating field. Since the accelerating bunch is short $[L(\tau)\ll\lambda_p]$ we can consider it as a plane bunch and find the normalized amplitude of the wakefield excited by the bunch from expression $E_{b,\max}=k_p(v_b/c)(N_b/n_p)$ [24], where v_b and N_b are velocity and the surface density of the bunch, correspondingly. This expression is valid both in linear and nonlinear regimes. In our case $N_b=n_{b0}L_0\delta/k_p$, where $\delta\leqslant 1$ is the ratio of trapped electrons number to the total number of particles in the initial bunch, and we have

$$E_{b,\max}=\delta(v_b/c)(n_{b0}L_0/n_p). \tag{7}$$

The normalized amplitude of moderately nonlinear laser wake wave, considering in this paper, is about unit. So, we can neglect the wakefield generated by the bunch if $E_{b,\max}\ll 1$, or when

$$n_{b0}\ll n_p(c/v_b)(1/L_0\delta).$$

For $n_p\sim 10^{16}$–10^{18} cm^{-3} (that is typical for the LWFA experiments [3]), $v_b\approx c$, $\delta\approx 1$, and the initial bunch length in order of λ_p ($L_0\sim 5-10$) this condition reads $n_{b0}<10^{14}$–10^{16} cm^{-3}. The density of the accelerating bunch may be in order of plasma density.

Thus, the one-dimensional analysis has showed the possibility of trapping, essential compression, and high-gradient acceleration of a low-energy electron bunch in moderately nonlinear laser wakefield.

III. TRAPPING, COMPRESSION, AND ACCELERATION IN LASER WAKEFIELD EXCITED IN A PLASMA CHANNEL

In this section we consider our scheme of LWFA for the case of laser wakefield excited in a plasma channel and study the peculiarities of radial motion of test electrons during trapping and acceleration.

A. Nonlinear laser wakefield excited in a plasma channel

As was mentioned in the Introduction, the plasma channel is necessary to guide a laser pulse. This allows us to essentially increase the laser-plasma interaction distance [12], that, in its turn, provides ultra-relativistic acceleration in the wakefield [3]. Nonlinear axially symmetrical laser wakefields excited in a plasma channel are described by the following system of equations [15]:

$$\beta\frac{\partial p_z}{\partial\xi}-\frac{\partial\gamma_e}{\partial\xi}-\beta^2E_z=0, \tag{8a}$$

$$\beta\frac{\partial p_r}{\partial\xi}-\frac{\partial\gamma_e}{\partial r}-\beta^2E_r=0, \tag{8b}$$

$$-\frac{\partial H_\theta}{\partial\xi}+\beta\frac{\partial E_r}{\partial\xi}+\beta_rN_e=0, \tag{8c}$$

$$\nabla_\perp H_\theta+\beta\frac{\partial E_z}{\partial\xi}+\beta_zN_e=0, \tag{8d}$$

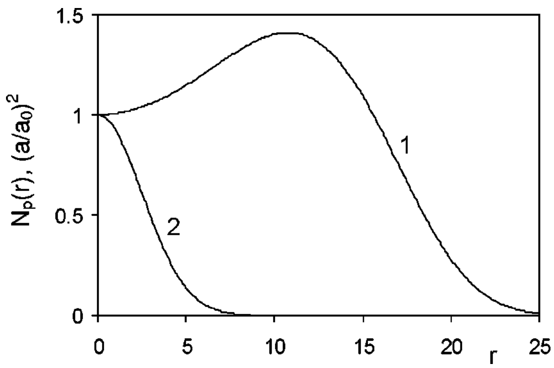

FIG. 6. The radial profiles of dimensionless unperturbed electron density in the plasma channel (curve 1) and the normalized laser-pulse intensity (curve 2), $r_{ch}=\sigma_r=5$, $b=0.01$.

$$\beta\frac{\partial H_\theta}{\partial\xi}-\frac{\partial E_r}{\partial\xi}+\frac{\partial E_z}{\partial r}=0, \tag{8e}$$

$$N_e=N_p(r)-\nabla_\perp E_r-\frac{\partial E_z}{\partial\xi}, \tag{8f}$$

where $E_{z,r}$ and H_θ are longitudinal and radial components of the electric field and azimuthal component of the magnetic field normalized to the on-axis wave-breaking field $E_{WB}(r=0)=m_e\omega_p(r=0)v_g/e$, $p_{z,r}$ are the normalized components of plasma electron momentum, $\gamma_e=(1+p_z^2+p_r^2+a^2/2)^{1/2}$ is the relativistic factor, $\beta_{z,r}=p_{z,r}/\gamma_e$, $N_e=n_e(\xi,r)/n_p(0)$ is the normalized density of plasma electrons, $n_p(r)$ is unperturbed plasma density in the channel, $N_p=n_p(r)/n_p(0)$, $\nabla_\perp=\partial/\partial r+1/r$. The force acting on the relativistic electrons in the wakefield is $\mathbf{F}(-eE_z,-e(E_r-\beta H_\theta),0)$. According to Eq. (8e)

$$\frac{\partial E_z}{\partial r}=\frac{\partial(E_r-\beta H_\theta)}{\partial\xi}\equiv-\frac{\partial f_r}{\partial\xi}. \tag{9}$$

So, the field of forces $\mathbf{F}$ is potential because $\nabla\times\mathbf{F}=0$, and one can write $\mathbf{F}=\nabla\Phi(\xi,r)$, here $\Phi=1-\int_\xi^0 E_z d\xi$.

In this section we consider an axially symmetric laser pulse which has Gaussian profile both in longitudinal and radial directions:

$$a(\xi,r)=a_0\exp[-(\xi-\xi_c)^2/\sigma_z^2]\exp(-r^2/\sigma_r^2).$$

The laser pulse is guided in preformed plasma channel which has the following unperturbed electron density:

$$N_p=\left(1+\Delta\frac{r^2}{r_{ch}^2}\right)\exp\left(-b\frac{r^4}{r_{ch}^4}\right), \tag{10}$$

where r_{ch}, Δ and $b\ll 1$ are constant values. Such a density profile is typical for plasma channels created in experiments [25]. Suppose that the pulse is guiding without change in its radius σ_r. In this case $\sigma_r=r_{ch}$ and $n_p(r_{ch})-n_p(0)=1/\pi r_e r_{ch}^2$, where $r_e=e^2/m_ec^2\approx 2.8\times 10^{-13}$ cm is the classical electron radius and all values are dimensional [12]. Then, in expression (10), $\Delta=(2/\sigma_r\beta_g)^2$.

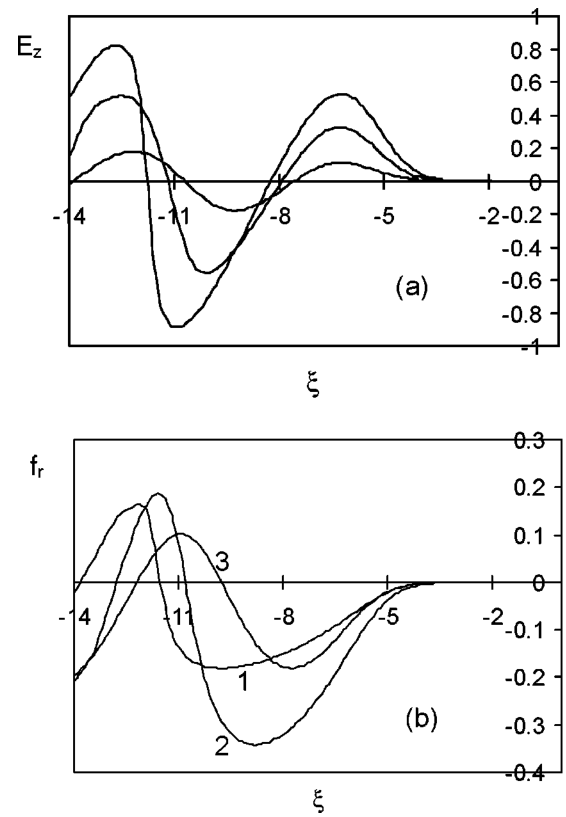

FIG. 7. The two-dimensional nonlinear laser wakefield excited in the plasma channel with the radial density profile shown in Fig. 6, $a_0=2$, $\sigma_z=2$, $\sigma_r=5$. (a) The longitudinal electric field for $r_0=0$, 3, and 5 in the order of magnitude reduction. (b) The focusing field $f_r=\beta_g H_\theta-E_r$. 1, $r=1$; 2, $r=3$; 3, $r=5$. All variables are normalized.

Equations (8a)–(8f) were solved numerically for the following parameters of the problem: $a_0=2$, $\sigma_z=2$, $\sigma_r=5$, and $\gamma_g=10$. In this case $\Delta\approx 0.16$, the value of b was chosen to be 0.01. In Fig. 6 we present the radial profile of unperturbed plasma density and the radial behavior of the normalized laser-pulse intensity, namely, $\exp(-2r^2/\sigma_r^2)$. Figure 7 shows the longitudinal electric field and the focusing field $f_r=\beta H_\theta-E_r$ of the wakefield excited. One can see that the wake wavelength decreases as r increases. This is caused by the radial increase of unperturbed plasma density in the channel [3,13] and by the nonlinear increase of wavelength with the wake wave amplitude which is at maximum on the axis [15,16,26]. Figure 7 shows also the nonlinear steepening of the accelerating field like the one that takes place in the one-dimensional wakefield (see Fig. 1). Due to the dependence of the wavelength on r, the field in the radial direction grows more chaotic as the distance from the laser pulse increases. In fact, the oscillations of the plasma for different r are started behind the pulse with nearly equal phases but different wavelengths. As the value of $|\xi|$ increases, the change of phase in the transverse direction becomes more and more marked. This leads to a curving of the phase front and to oscillations in the transverse direction [15,16,26]. Such behavior of the wakefield excited in a plasma channel leads to the transverse multistream motion of plasma elec-

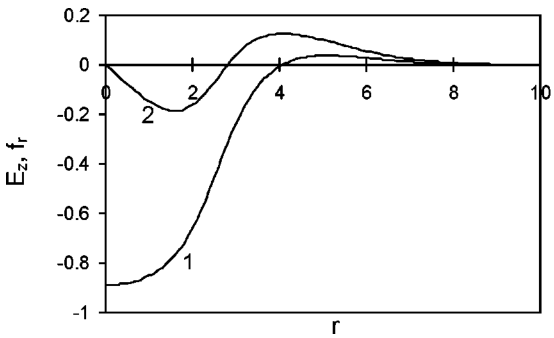

FIG. 8. The radial behavior of the wakefield shown in Fig. 7, at $\xi=-10.9$. 1, dimensionless longitudinal electric field $E_z(\xi=-10.9,r)$; 2, the normalized focusing force $f_r(\xi=-10.9,r)$.

trons in the wake and to the transverse wave breaking [27]. The radial dependence of the longitudinal electric field and the focusing force is shown in Fig. 8 for point $\xi=-10.9$ at which the on-axis accelerating field reaches its maximum. We see that the wakefield changes its sign and is steepened. For the ultra-relativistic acceleration of electrons one needs to use a region in the wakefield where the conditions $E_z<0$ and $f_r<0$ are satisfied simultaneously. The radial steepening leads to the radial restriction or the region suitable for acceleration. Near the first accelerating maximum of the wakefield shown in Fig. 7, the suitable region is $r<2.8$. As the distance from the laser pulse increases, the suitable region becomes narrower, so that at some distance the wakefield is highly irregular. Thus, the most preferable for electron acceleration is the region of the first accelerating maximum in the wake.

B. Equation of motion of bunch electrons

Three-dimensional vector equation of motion of bunch electrons is

$$\frac{d\mathbf{p}}{d\tau}=-\beta_g(\mathbf{E}+\boldsymbol{\beta}\times\mathbf{H})-\frac{1}{4\beta_g\gamma}\nabla a^2. \tag{11}$$

Here all values are dimensionless, $\boldsymbol{\beta}=\mathbf{v}/c=\mathbf{p}/\gamma$ is the normalized velocity, $\gamma=(1+\mathbf{p}^2+a^2/2)^{1/2}=[(1+a^2/2)/(1-\beta^2)]^{1/2}$ is the relativistic factor. For the momentum components, from Eq. (11) one has

$$\frac{dp_r}{d\tau}=-\beta_g(E_r-\beta_z H_\theta)-\frac{1}{4\beta_g\gamma}\frac{\partial a^2}{\partial r}, \tag{12a}$$

$$\frac{dp_\theta}{d\tau}=0, \tag{12b}$$

$$\frac{dp_z}{d\tau}=-\beta_g(E_z+\beta_r H_\theta)-\frac{1}{4\beta_g\gamma}\frac{\partial a^2}{\partial \xi}. \tag{12c}$$

It follows from Eq. (12b) that the azimuthal momentum is conserved, $p_\theta=p_\theta(0)=\text{const}$, $\beta_\theta(\tau)=p_\theta(0)/\gamma(\tau)$. The azimuthal momentum has no essential influence on the axial and radial dynamics, and we suppose in this paper that $p_\theta(0)=0$. Equation (11) gives the following equation for the energy of electrons:

$$\frac{d\gamma}{d\tau}=-\beta_g(\boldsymbol{\beta}\cdot\mathbf{E})-\frac{1}{4\gamma}\frac{\partial a^2}{\partial\xi}. \tag{13}$$

From Eqs. (12), (13), and (9) we obtain the integral of motion

$$\gamma-\beta_g p_z-\Phi(\xi,r)=\text{const}, \tag{14}$$

which formally coincides with the one-dimensional integral of motion (3) [14,28,29]. Electrons can be trapped in the region where wakefield is both accelerating and focusing. For the scattered particles, from Eq. (14) one has $p_r^2=(S+\beta_g p_z)^2-p_z^2-1$, here $S=[1+\mathbf{p}^2(0)]^{1/2}-\beta_g p_z(0)$. If an electron is initially nonrelativistic ($|\mathbf{p}(0)|\ll 1$, $S\approx 1$), then $p_r\approx(2p_z)^{1/2}$ and $\tan\theta=p_r/p_z\approx[2/(\gamma-1)]^{1/2}$, where θ is the angle between the z axis and final momentum of the scattered electron [30].

Taking into account Eq. (13), we rewrite Eqs. (12a) and (12c) in the form

$$\frac{d^2\xi}{d\tau^2}+\frac{1}{\gamma}[(1-\beta_z^2)E_z-\beta_z\beta_r E_r+\beta_r H_\theta]+\frac{(1-\beta_g\beta_z)}{4\beta_g^2\gamma^2}\frac{\partial a^2}{\partial\xi}=0, \tag{15a}$$

$$\frac{d^2 r}{d\tau^2}+\frac{1}{\gamma}[(1-\beta_r^2)E_r-\beta_z\beta_r E_z-\beta_z H_\theta]-\frac{1}{4\beta_g\gamma^2}\left(\beta_r\frac{\partial a^2}{\partial\xi}-\frac{1}{\beta_g}\frac{\partial a^2}{\partial r}\right)=0. \tag{15b}$$

The normalized components of velocity obey the formulas $\beta_z=\beta_g(1+d\xi/d\tau)$ and $\beta_r=\beta_g dr/d\tau$. For particles trapped in the wakefield, we suppose that during acceleration $\beta_z\approx 1$, $\beta_r^2\ll 1$ and $r<1$ (the numerical results presented below show that this is the case). Then, from Eq. (15a) one has

$$d^2\xi/d\tau^2\approx E_z/\gamma^3. \tag{16}$$

It follows from this equation that $d\gamma/d\tau\approx -E_z$ and $\gamma\approx-\int E_z d\tau$. Thus, the longitudinal dynamics of accelerating particles is approximately the same as in the one-dimensional case. The radial motion of electrons, according to Eq. (15b), obeys the equation

$$\frac{d^2 r}{d\tau^2}+\frac{|E_z|}{\gamma}\frac{dr}{d\tau}+\Omega^2 r\approx 0, \tag{17}$$

where $\Omega=(|\partial f_r/\partial r|/\gamma)^{1/2}$ is the betatron frequency. Supposing that the value of E_z is approximately conserved during acceleration, we can write $\gamma\approx|E_z|(\tau-\tau_{tr})$. In this case solution of Eq. (17) is

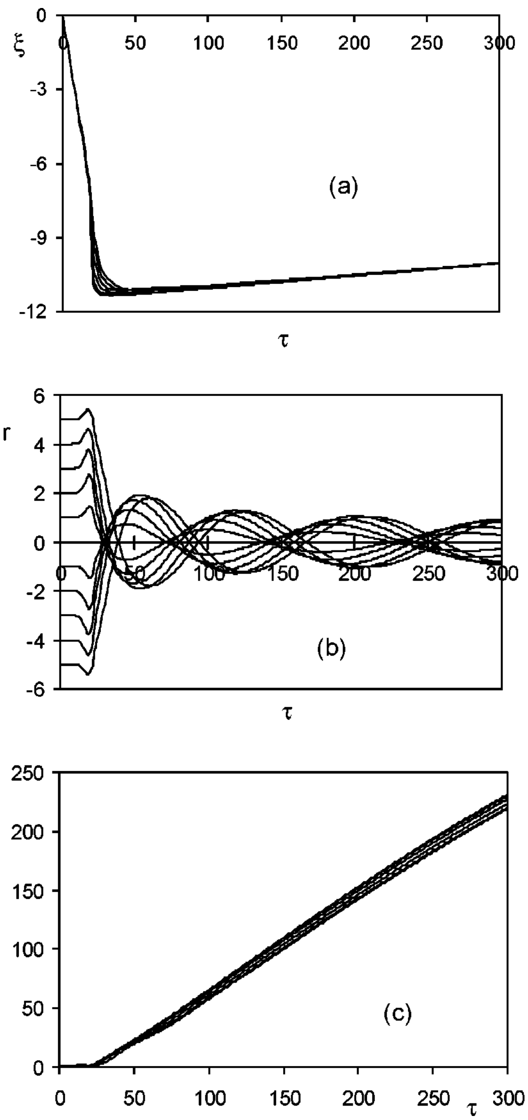

FIG. 9. Trapping and acceleration of electrons with zero initial momenta in the wakefield given in Fig. 7, $p_{z0}=0.8$, $\xi_0=0$. Longitudinal (a) and radial (b) positions and relativistic factor (c) of electrons. The coordinates and time are normalized.

$$r=r(\tau_{tr})J_0(2[|\partial f_r/\partial r|(\tau-\tau_{tr})/|E_z|]^{1/2}), \tag{18}$$

where J_0 is the Bessel function of zero order.

C. Results of test-particle simulations and discussion

Motion of test electrons in the two-dimensional (2D) wakefield presented in Fig. 7 was investigated by numerical solution of exact Eqs. (15a) and (15b) for different initial positions and momenta. Figure 9 shows the behavior of electrons with zero initial transverse momenta and with different initial radial positions. One can see that particles are trapped near the first accelerating maximum in the wake. During the trapping, electrons concentrate near the axis due to the focusing force $\beta_z H_\theta - E_r$. Since the longitudinal size of the trapped bunch is much less than the plasma wavelength and its transverse size is essentially less than that of the laser pulse, the electrons experience approximately the same accelerating field. Therefore, the longitudinal dynamics of the electrons is well described by the one-dimensional theory. The focusing force acting on the bunch electrons depends on r linearly (see Fig. 8). The small bunch sizes (as compared with the wakefield characteristic sizes) and the fact that electrons are trapped near the accelerating maximum provide high-accelerating gradient and relatively small energy spread. For example, the relative energy spread of electrons presented in Fig. 9 is $\approx 5\%$ at $\tau=300$. The numerical results show that dynamics of the accelerating bunch is well described by approximate Eqs. (16)–(18). The betatron oscillations of the accelerating electrons are clearly seen in Fig. 9(b). The wavelength of this oscillation decreases with the increase of the particle's energy that conform to the formula for betatron frequency. Radial velocity of accelerating electrons is much less than the longitudinal one, $|\beta_r(\tau)|<0.1 \ll \beta_z(\tau)$. One can see also that even electrons which are initially at the periphery $[r(\tau=0)\equiv r_0 \sim \sigma_r]$ can be trapped in the wakefield and accelerated. The characteristic dependence of the minimum trapping threshold $p_{z,\min}$ on the initial radial position of electron is presented in Fig. 10. Figure 11 shows the minimum and maximum initial radial momenta of trapped electrons in dependence on initial radial position. The figure witnesses that electrons which initially move at a relatively high angle to the axis (up to 10°) also can

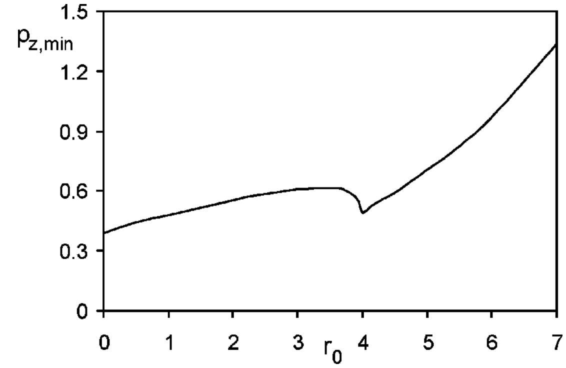

FIG. 10. The characteristic dependence of the minimum trapping threshold on the initial radial position of the electron, $p_{r0}=0$, $\xi_0=0$. Both variables are dimensionless.

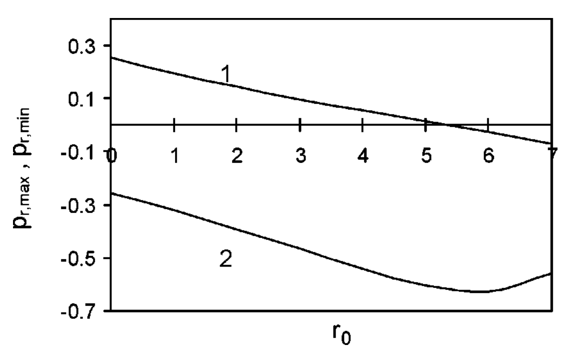

FIG. 11. Maximum (curve 1) and minimum (curve 2) dimensionless initial radial momenta of trapped electrons depending on the normalized initial radial position, $p_{z0}=0.8$, $\xi_0=0$.

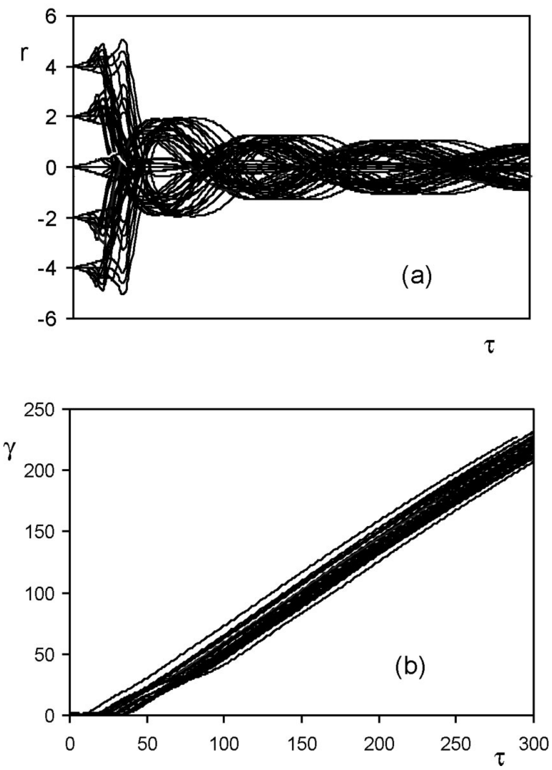

FIG. 12. Trapping, compression, and acceleration of an electron bunch in the wakefield presented in Fig. 7. Initial parameters of the bunch are: $0 \leqslant \xi_0 \leqslant 5$, $r_0 \leqslant 4$, $0.6 \leqslant p_{z0} \leqslant 0.8$, $-0.02 \leqslant p_{r0} \leqslant 0.02$. The normalized radial positions (a) and relativistic factor (b) of electrons.

be trapped and accelerated. This again is caused by the focusing force and the fact that electrons are initially nonrelativistic ($\gamma_0 \sim 1$).

In Fig. 12 we show behavior of electrons of a bunch with the following initial parameters: $0 \leqslant \xi_0 \leqslant 5$, $r_0 \leqslant 4$, $0.6 \leqslant p_{z0} \leqslant 0.8$, and $-0.02 \leqslant p_{r0} \leqslant 0.02$. The passing particles (not showed) are well separated from accelerating one both spatially and energetically. The length of the accelerating bunch in this case also is much less than the plasma wavelength $[L(\tau=100) \approx 0.27,\ L(\tau=300) \approx 0.19]$. The radius of the bunch R decreases relatively slowly during acceleration and is essentially less than the characteristic transverse size of the wakefield σ_r, $R(\tau) \sim 1$; the bunch radius can be reduced by the choice of smaller laser spot size. The absolute energy spread does not change practically, $\Delta\gamma \approx 24$, but the relative energy spread falls and is equal to about 10% at $\tau = 300$. Our simulations show that neither initial transverse size of the bunch nor initial radial momenta of trapped electrons have any influence essentially on the energy spread and dynamics of accelerating bunch. The estimations of absolute and relative energy spreads presented in Sec. II are valid also in the 2D case.

The total number of electrons trapped and their density can be estimated from expressions

$$N_{tot} \sim \pi n_{b0} \sigma_r^2 L_0 \delta / k_p^3, \tag{19}$$

$$n_b \sim n_{b0} \delta (\sigma_r / R)^2 (L_0 / L). \tag{20}$$

The on-axis amplitude of the linear wake wave excited by the bunch is reduced by the factor $T(R) = 1 - R K_1(R) < 1$ [31] (where K_1 is the modified Bessel function) as compared to the one-dimensional case (see Sec. II). Therefore, in our case, for the amplitude of wakefield generated by the bunch, we have $E_{b,\max} \approx T L n_b / n_p$. This wakefield can be neglected when $E_{b,\max} \ll E_{z,\max}$, or taking into account Eq. (20)—if $T L_0 \delta (\sigma_r / R)^2 (n_{b0} / n_p) \ll 1$; when $R \ll 1$, $T \approx R^2/2$, and this condition reads $L_0 \sigma_r^2 n_{b0} \delta / 2 n_p \ll 1$. The total number of trapped electrons, according to Eq. (19), is restricted by the following condition: $N_{tot} \leqslant \pi n_p k_p^{-3} (R^2/T) \approx 1.4 \times 10^7 (R^2/T) \lambda_p$, with λ_p in μm.

For the normalized emittance $\varepsilon_n = \sigma_0^2 / \beta$ (here σ_0 is the matched transverse size of the bunch, β is the betatron length) of the accelerating bunch, in our notations, one can write $\varepsilon_n \sim R^2 \Omega \lambda_p / 4\pi^2$. In the case $\lambda_p = 100\ \mu$m ($n_p \approx 10^{17}$ cm^{-3}), for the bunch presented in Fig. 12, $\varepsilon_n \sim 8$ nm$/\gamma^{1/2}$; for example, $\varepsilon_n \sim 0.5$ nm when $\gamma = 300$, that is comparable with the emittance expected in the TeV-range laser wakefield accelerator [32,33] (see also Refs. [14,33–36] for the dynamics of the accelerating bunch).

IV. SUMMARY

The results of the present paper show the possibility of trapping, essential compression both in longitudinal and transverse directions, and ultra-relativistic acceleration of an initially nonrelativistic or weakly relativistic electron bunch in moderately nonlinear ($a_0 \sim 1$, $E_{z,\max} \sim 1$) laser wakefield. The initial bunch can be generated, for example, by a photocathode. So far as the electron bunch is initially nonrelativistic ($\gamma_0 \sim 1$), trapping and compression take place during time interval comparable with the plasma wave period, that is much less than the time scale of longitudinal dynamics of relativistic particles in the wake [35]. Due to the fact that trapped bunch sizes are essentially less than characteristic spatial scales of the wake wave, the energy spread in the accelerated bunch can be relatively low, namely, a few percent. In our scheme the problems connected with the wake wave front curvature also are removed. The electron bunch trapped and accelerated can be accelerated further in the multistage LWFA [33].

Thus, the scheme of LWFA proposed, has the following advantages: (a) instead of injection of an enough dense relativistic electron bunch with small sizes (in order of a micrometer), our scheme utilizes a nonrelativistic, rare and long electron bunch, that is much easier to get technically, (b) femtosecond electron bunch synchronization in the laser wakefield is not required, (c) effective electron bunch compression, and (d) spatial and energetic separation of the initial electrons, that can decrease the trapped bunch emittance.

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